Exercise 2: $(1+2k)x^2-10x+2=0$, with $k\in\mathbb{R}$. Equal Roots?

Cases revolving around Δ - a.k.a. the descriminant

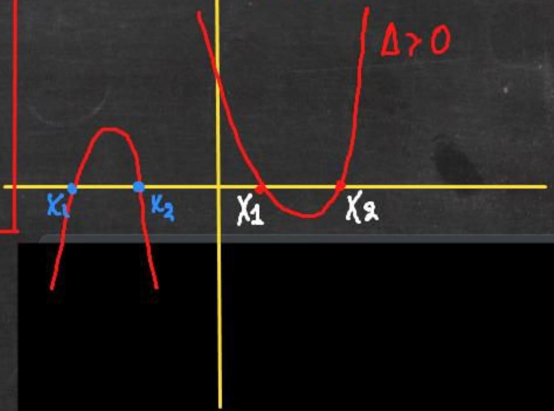
1) If $\Delta>0$ then we have 2 distinct - non equal roots
2) If $\Delta=0$ then we have 1 real solution which is considered "double"
3) If $\Delta<0$ then we have no real roots

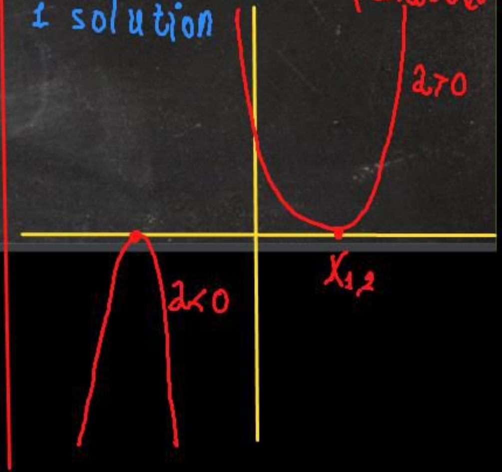
$\Delta = b^2 - 4ac$

In order to have 2 equal roots we need the descriminant to be equal to 0. Therefore, we're gonna solve the equation Δ = 0

$\Delta = 0 \Rightarrow (-10)^2 - 4\cdot(1+2k)\cdot 2 = 0 \Rightarrow 100 - 8\cdot(1+2k) = 0 \Rightarrow$

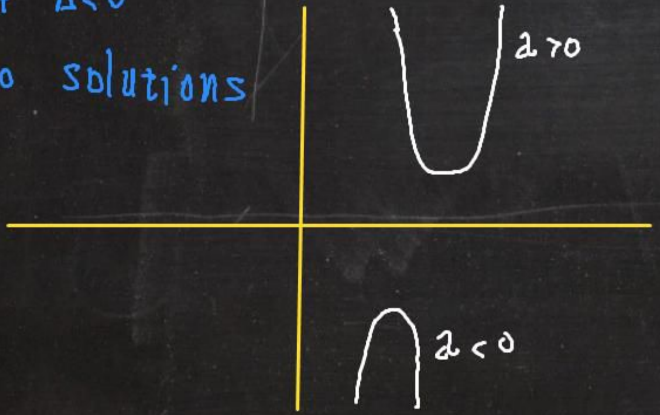
$100 - 8 - 16k = 0 \Rightarrow \frac{92}{16} = \frac{16k}{16} \Rightarrow \boxed{k = 5.75}$





If Δ<0

no solutions



Exercise 2: Consider the equation $(1+2k)x^2 - 10x + 2 = 0$, with $k \in \mathbb{R}$. For which values of k does the function have *equal roots*?

Composite Function :

1. $f(x)$ and $g(x)$

$$\left.\begin{aligned} f(x) &= x+2 \\ g(x) &= \sqrt{x} \end{aligned}\right\} \quad \begin{aligned} f(g(x)) &= g(x)+2 \\ f(g(x)) &= \sqrt{x}+2, \\ fog(x) &= \sqrt{x}+2, \end{aligned}$$

$x \geq 0 \Rightarrow D_{fog} = [0, +\infty)$

Therefore, in order to carefully set a composite function we have not only to substitute the appropriate function inside, but at the same time we have to take care of the domain of the composite function as well

2. $f(x) = x^3 - 2\cdot x^2 + x$, $D_f = R$ (polinomial)

$g(x) = \sin x$, $D_g = R$

$$\boxed{g(f(x)) = gof(x) = \underset{\text{structure}}{\sin}(x^3 - 2x^2 + x)} \quad D_{gof} = R$$

The domain of the function, refers to the values that x can take, when we consider every possible restriction that we have in the formula of the function

3. $f(x) = \frac{1}{x}$, $g(x) = x - 2$ $\quad D_g = R$

$\boxed{x \neq 0}$

$D_f = (-\infty, 0) \cup (0, +\infty)$

$fog(x) = f(g(x)) = \frac{1}{g(x)} \Rightarrow$

$f(g(x)) = \frac{1}{x-2}$, $D_{fog} = (-\infty, 2) \cup (2, +\infty)$

$x - 2 \neq 0$

$\boxed{x \neq 2}$

Exercise 5

5 points out of 100

(i) $2x^2-8x+9$ express it in the form $a(x+b)^2+c$, where $a,b,c \in \mathbb{Z}$

(ii) Given that $f(x)=x-2$ and $gof(x)=2x^2-8x+9$, find $g(x)$.

Quadratic + Composite

Step 1 : What kind of exercise do we have here ?

(i)

Step 2 :

$$2\cdot(x^2-\underbrace{4x}+4)+1=2\cdot(x-2)^2+1 \quad \begin{cases} a=2\\ b=-2\\ c=1\end{cases}$$

$(a-b)^2=a^2-2ab+b^2$

Do those calculations

$x^2-\underbrace{2\cdot x\cdot 2}_{4x}+2^2$

In this kind of exercises you have to be good in factorization

(ii)

$gof(x)=\underbrace{2x^2-8x+9}\Rightarrow$

$g(f(x))=2\cdot(x-2)^2+1$

$g(x-2)=2\cdot(x-2)^2+1$

Set $x-2=u \Rightarrow g(u)=2\cdot u^2+1$ (1 variable)

Substitute again u with x

$\boxed{g(x)=2x^2+1}$

Exercise 5:

i) Express $2x^2-8x+9$ in the form $a(x+b)^2+c$, where $a,b,c\in\mathbb{Z}$

ii) Given that $f(x)=x-2$ and $gof(x)=2x^2-8x+9$, find $g(x)$.