

Gravitational Field:

$$g = \frac{N}{kg}$$

gravitational Field Strength

(gravitational force)

$$F = G \cdot \frac{M \cdot m}{r^2}$$

(coulomb force)

$$F_c = K \cdot \frac{q_1 \cdot q_2}{r^2}$$

$$G = 6.67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$$

gravitational constant

$$G = \frac{F \cdot r^2}{M \cdot m}$$

Since the gravitational force can be equal to gravitational attractive force, we can derive that:

$$F_g = F_{g_{attr}} \Leftrightarrow m \cdot g = G \cdot \frac{m \cdot M}{r^2} \Leftrightarrow$$

$$g = G \cdot \frac{M}{r^2}$$

This relationship indicates that the gravitational field is proportional to the mass of the large object and inversely proportional to the square of its radius.

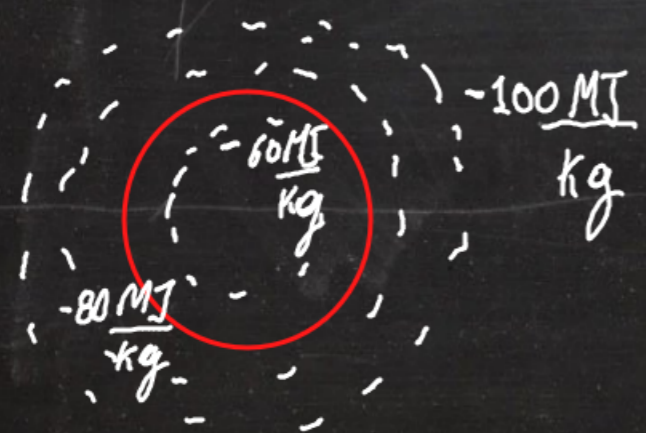
$$V_g = - \frac{GM}{r} \text{ (gravitational potential)}$$

infinite

So in order to bring an object from infinity to a point P in space (which is the gravitational potential) you consume energy (the object is supposed to have energy stored at infinity let it be kinetic energy for example)

$$V_{potential} = \frac{W}{q}$$

$$V_g = - \frac{GM}{r}$$



2 definitions can be given about Gravitational potential (and in comparison to the electric field electric potential):

- 1) One of them talks about energy / kg (or energy / coulomb - per charge)**
- 2) The other talks about the consumption of energy in order to bring the point mass or point charge from one point in space to another.**

Gravitational Fields

$$g = -\frac{GM}{r}$$

$$g = -\frac{\Delta V_g}{\Delta r}$$

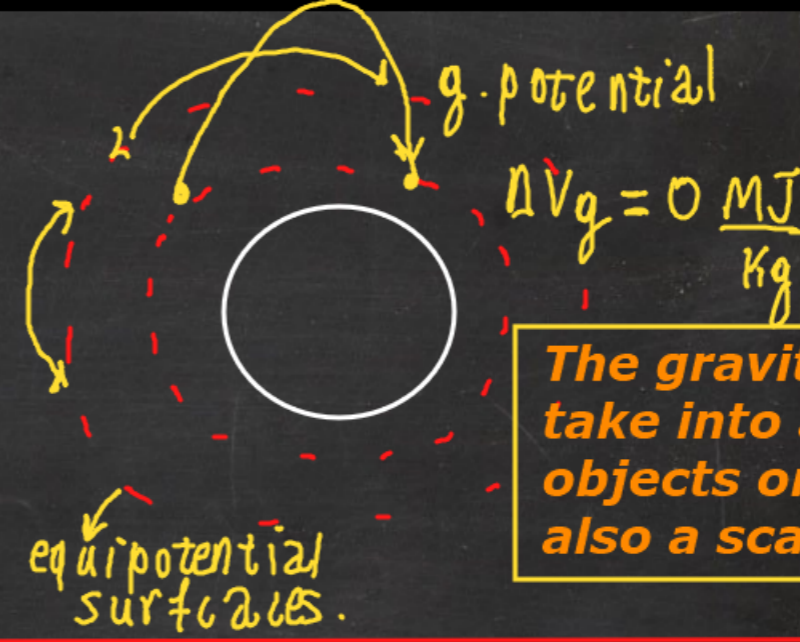
The difference in the gravitational field strength in a specific point or surface outside of the planet

(from mechanics: $m \cdot g \cdot h$)

$$E_p = m \cdot V_g = -m \cdot \frac{G \cdot M}{r} = -G \cdot \frac{m \cdot M}{r}$$

$$F_g = G \frac{m \cdot M}{r^2}$$

$F_g \cdot r = E_p$
 $F \cdot x = W$
 ☺ we lur physics



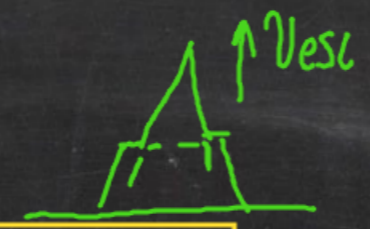
The gravitational does not take into account the objects orbit (since it is also a scalar quantity)

If we apply conservation of mechanics in combination other rules as well we can derive a few other formulas for planetary and orbital motion:

$$V_{esc} = \sqrt{\frac{2GM}{r}}$$

(proportional to square root of mass, inversely proportional to the square root of radius)

$$g = -\frac{GM}{r} \left(\frac{N}{Kg} \right)$$



So if the radius of a large mass object becomes greater this means that our field gravitational will become less, leading to a smaller magnitude of gravitational, therefore the escape wont have to be that large.

So mass has a greater priority when we are trying to assess the gravitational field strength of a planetary object compared to its radius.

Relationships between circular motion and gravitational fields:

$$F_{\text{cen.}} = F_{\text{grav.}} \Leftrightarrow \frac{m \cdot v^2}{r} = G \cdot \frac{Mm}{r^2}$$

Mass does not play a role to the orbital of an object when you are in VACUUM !!!

$m = \text{mass of the object}$

$$F_c = k \cdot \frac{q_1 \cdot q_2}{r^2}$$

$$F_g = G \cdot \frac{M \cdot m}{r^2}$$

$$v^2 = \frac{GM}{r} \Rightarrow$$

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$$

$$v_{\text{orbital}} = \omega \cdot r \Rightarrow$$

$$\omega \cdot r = \sqrt{\frac{GM}{r}} \Leftrightarrow$$

$$\omega^2 \cdot r^2 = \frac{GM}{r} \Rightarrow \omega^2 = \frac{GM}{r^3} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3} \Rightarrow$$

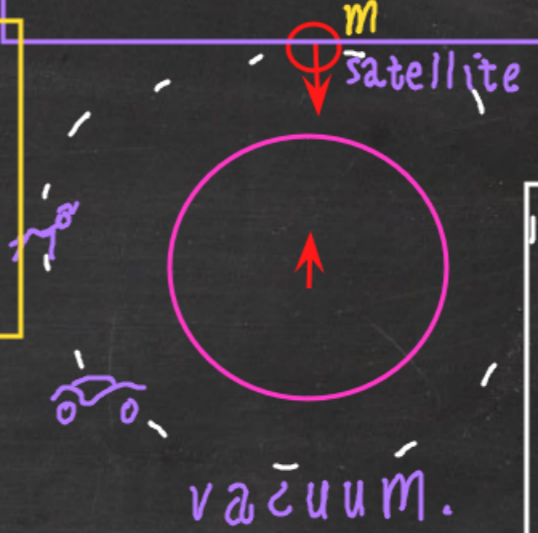
$$\frac{4\pi^2}{T^2} = \frac{GM}{r^3} \Leftrightarrow$$

$$T^2 \cdot GM = 4\pi^2 r^3 \Rightarrow$$

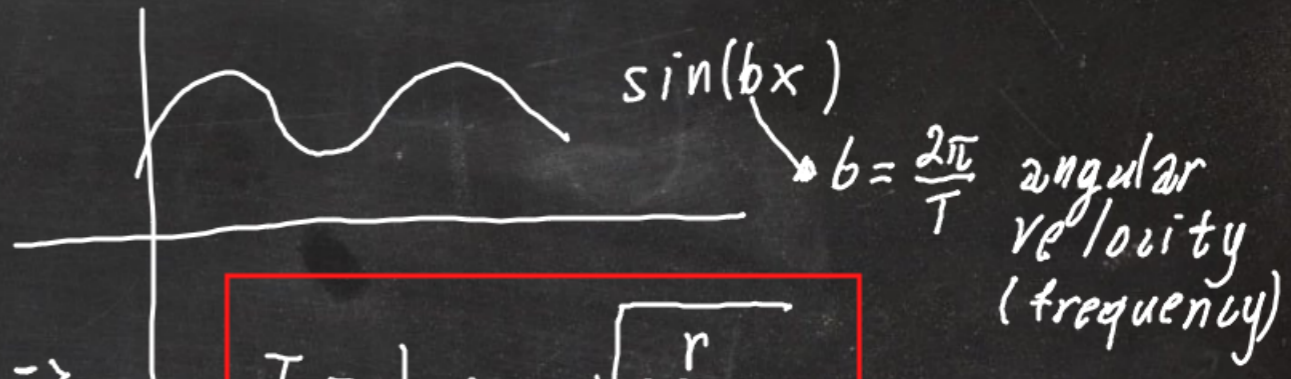
$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 \cdot r^3}{GM}} = 2\pi \cdot r \sqrt{\frac{r}{GM}}$$

$$T = T_{\text{circ.}} \cdot \sqrt{\frac{r}{GM}}$$



The coulomb force between 2 charges can be both attractive or repelling. The gravitational force is only attractive.



Mathematical that mass of an object doesn't play a role in its orbital period.

Background knowledge:

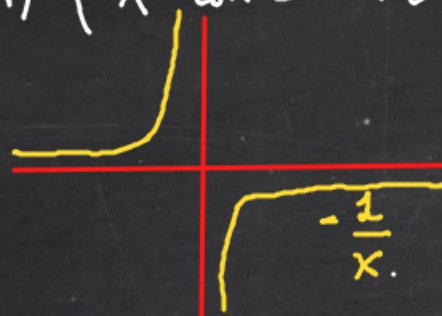
$$f(x) = \frac{1}{x} \text{ (hyperbola)}$$

$$f(x) = \frac{1}{x}$$

$y=0$ (horizontal asymptote) $+\infty$

$f(x) \rightarrow -f(x)$ (x-axis reflection).

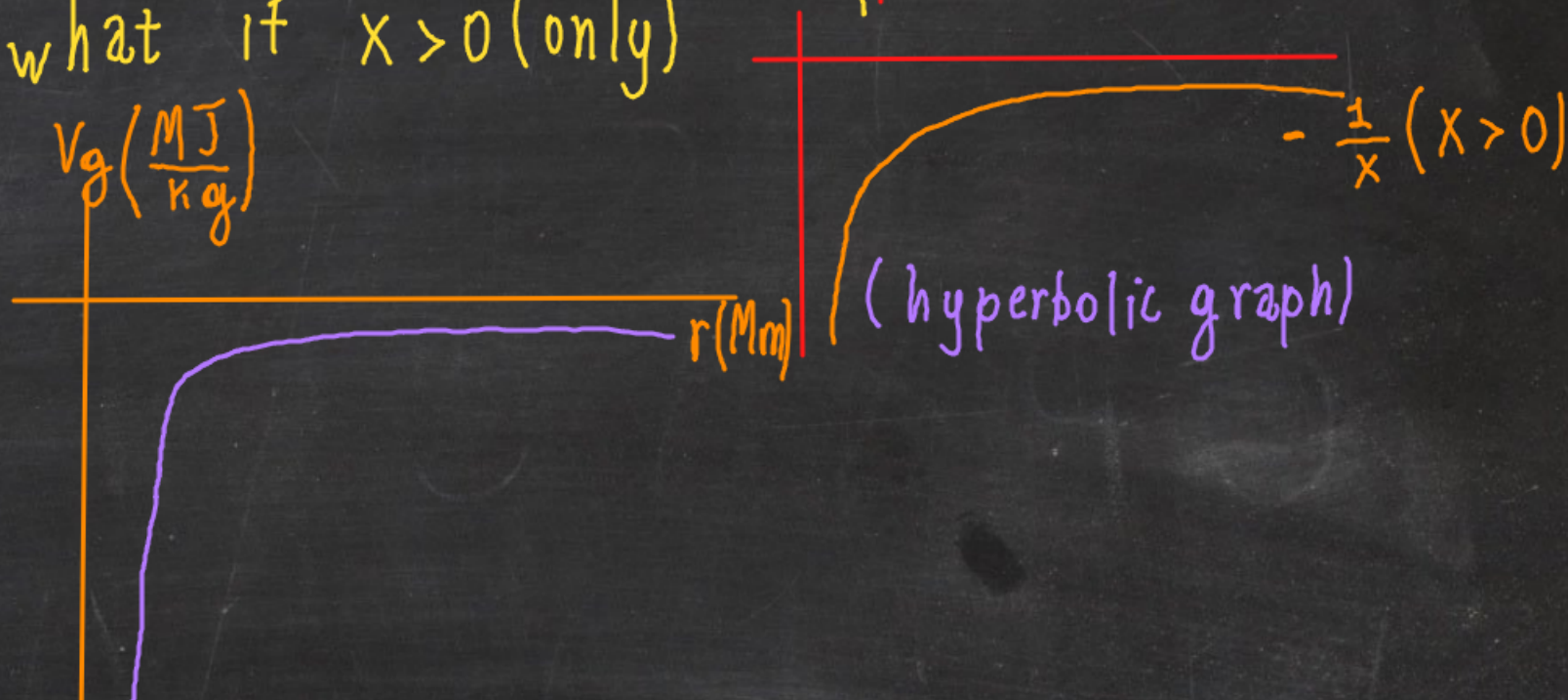
$$-f(x) = -\frac{1}{x}$$



what if $x > 0$ (only)

$$V_g \left(\frac{MJ}{kg} \right)$$

$$V_g = -GM \cdot \frac{1}{r} = -\frac{GM}{r}$$



(hyperbolic graph)

$$E_{\text{mech}} = E_{\text{Total}} = E_T = E_p + E_k$$

$$-\frac{GMm}{r} + \frac{1}{2} m \cdot \left(\sqrt{\frac{GM}{r}} \right)^2$$

$$-\frac{GMm}{r} + \frac{1}{2} \cdot \left(\frac{GMm}{r} \right) =$$

$$\frac{G \cdot Mm}{r} \left(-1 + \frac{1}{2} \right) =$$

$$-\frac{GMm}{2r}$$

17.3 Motion in a gravitational field

Orbital motion

A satellite of mass m orbits a planet of mass M with speed v . The radius of the orbit is r (Figure 17.26). The total energy E_T of this system is the sum of the kinetic energy E_k and the gravitational potential energy E_p .

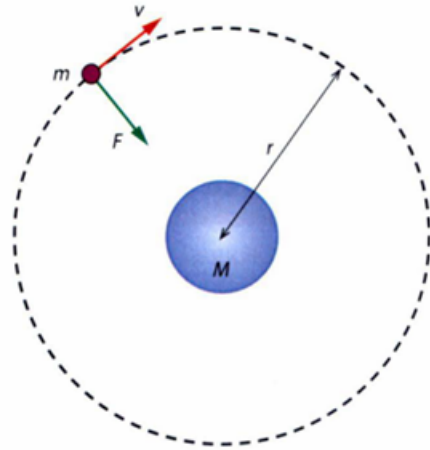


Figure 17.26: A system of a satellite orbiting a planet.

$$E_k = \frac{1}{2}mv^2 \text{ and } E_p = -\frac{GMm}{r}$$

So:

$$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Note that we do not include any kinetic energy for the planet, as we assume it does not move.

Orbital speed is given by:

$$v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

This means that the kinetic energy E_k is:

$$E_k = \frac{GMm}{2r}$$

The total energy of the system becomes:

$$E_T = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

The total energy of an orbiting satellite is $E_T = -\frac{GMm}{2r}$.

The minus tells us that there is energy loss in order to maintain the object into planetary orbit 373 >