$m = 0.4$ kg

$T_{in} = 80°C$

$Q = m \cdot c \cdot \Delta T$

$0.4 \cdot c \cdot 58 =$

$\boxed{23.2 \cdot c}$

0 °C. If this thermal energy is transferred to ice, it will cause the ice to melt. If thermal energy transfers back into water at 0 °C, the process repeats. The result is that more and more ice melts and the overall equilibrium temperature stays constant at 0 °C.

The method of mixtures

The electrical method described in worked example 7.5 is one method for measuring specific heat capacity and latent heat. Another method, the method of mixtures, measures the specific heat capacity of a solid as follows. A solid is put in a container of hot water and allowed time to reach a constant temperature. The temperature of the solid is thus that of the water and is recorded. The solid is then transferred into a calorimeter of known specific heat capacity and initial temperature which contains a liquid such as water (Figure 7.7). The calorimeter is insulated. The final temperature of the water is recorded after thermal equilibrium has been reached. Thermal equilibrium means that the temperatures of the bodies involved are the same.

For example, consider a mass of 0.400 kg of a solid at 80 °C that is put in a 100 g copper calorimeter containing 800 g of water at 20 °C. The final temperature of the water is measured to be 22 °C. From these values, we may deduce the specific heat capacity of the solid as follows.

Using $Q = mc\Delta T$, the amount of thermal energy (in joules) lost by the solid is:

$$0.400 \times c \times (80 - 22) = 23.2c$$

The amount of thermal energy gained by the calorimeter (see Table 7.1 for the value of c for copper) and the water is:

$$\underbrace{0.100 \times 385 \times (22 - 20)}_{\text{calorimeter}} + \underbrace{0.800 \times 4200 \times (22 - 20)}_{\text{water}} = 6797 \text{ J}$$

Equating this to the thermal energy $23.2c$ lost by the solid, we find that $c = 293 \text{ J kg}^{-1} \text{ K}^{-1}$.

EXAM TIP

It is likely that some thermal energy (heat) was dissipated to the surrounding air while it was being transferred. This means that the actual temperature of the solid is less than we supposed. The actual specific heat capacity is, therefore, larger than the calculated value.

The same method can be applied to measure the specific latent heat of fusion of ice. To do this, place a quantity of ice at 0 °C (the ice must therefore come from a mixture with water at 0 °C) into a calorimeter containing water at a few degrees above room temperature. Blot the ice dry before putting it into the calorimeter. The mass of the ice can be determined by weighing the calorimeter at the end of the experiment.

For example, suppose that 25.0 g of ice at 0.00 °C is placed in an aluminium calorimeter of mass 250 g containing 300 g of water at 24.0 °C. The temperature of the water is measured at regular intervals of time until the temperature reaches its minimum value of

So the basic idea behind the method of mixtures is that :

What you lose in thermal energy is what an other substance gains, example ->

Energy lost by the substance = Energy gained by the water and the calorimeter

$Q_{LOST} = Q_{gained} \Leftrightarrow$

GDC Solve $\left(23.2 \cdot C_{sub.} = 0.1 \cdot 385 \cdot 2 + 0.8 \times 4{,}200 \cdot 2\right)$

$Q_{s-w} + Q_{ice-w} \approx 0\,J$

conservation of energy.

at thermodynamics.

a Determine the energy required to raise the temperature of the radiator–water system by 1 K.

b If energy is provided to the radiator at the rate of 450 W, calculate how long it will take for the temperature to increase by 20.0 °C.

14 How much ice at −10 °C must be dropped into an aluminium cup containing 300 g of water

20 Steam at 100 °C is mixed with 200 g of ice at 0 °C to produce water at 50 °C. What is the mass of steam required?

21 Two bodies (same mass and initial temperature) are dropped into two identical containers filled with water. The mass and temperature of the water in both containers is the same. The graphs in Figure 7.9 show the cooling

20

$$m_{ice}\cdot \underline{L_f} + m_{ice}\cdot C_w\cdot \Delta T + \underline{m_s\cdot L_v} + m_s\cdot C_w\cdot(50-100) = 0\,J \iff$$

$$\underbrace{0.2\cdot 3.34\cdot 10^5 + 0.2\cdot 4200\cdot 50}_{} + m_s\cdot 2.26\cdot 10^6 + m_s\cdot 4200\cdot(-50) = 0\,J \iff$$

$$108{,}800 + 2.26\cdot 10^6 m_s - 210{,}000 m_s = 0 \iff$$

fusion = (Both s → li and li → s)

$$2{,}050{,}000\cdot m_s = -108{,}800$$

$$m_s = \left|\frac{-108{,}800}{2{,}050}\right| = 53g$$

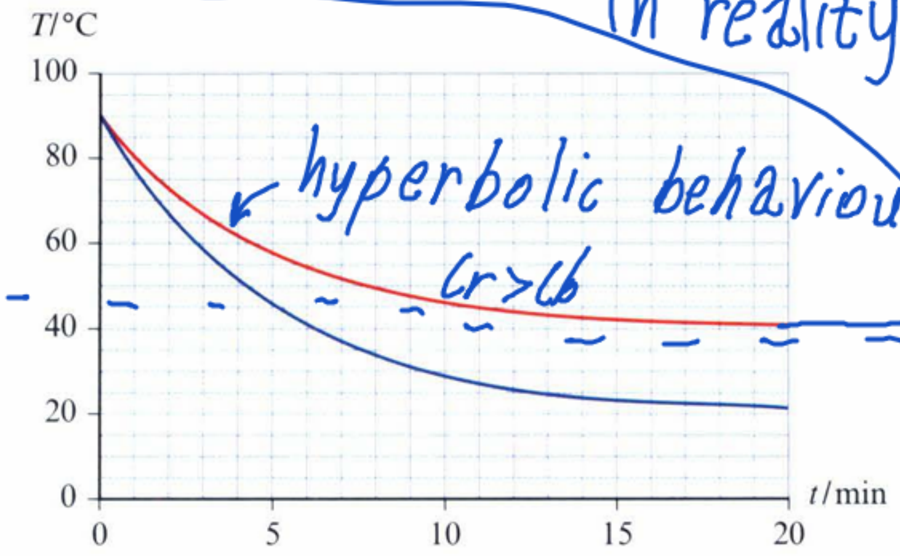
$$\boxed{m_s = 0.05\,Kg = 53\,g}$$

[redacted]

mass of steam required?

21 Two bodies (same mass and initial temperature) are dropped into two identical containers filled with water. The mass and temperature of the water in both containers is the same. The graphs in Figure 7.9 show the cooling curves of the two bodies.

In reality



hyperbolic behaviour

$c_r > c_b$

Figure 7.9: For question 21.

Which body has the higher specific heat capacity?

hyperbola

$y = \frac{a}{x}$

$y = \frac{ax+b}{cx+d}$

$y = c$ horizontal asympote

because we have reached a thermal equilibrium.

The object which is represented by the red curve has the higher specific heat capacity, since at the same time interval, it is able to maintain a higher temperature.

Thermodynamics:

- $Q = W + \Delta U$ (Heat = Work Produced / consumed + Change in Internal Energy)
- $\rho = \frac{m}{V}$ $\left(\text{density} = \frac{\text{mass}}{\text{volume}}\right)$
- * $\overline{E_K} = \frac{3}{2} \cdot K_B \cdot \Delta T$ (average Kinetic Energy per molecule)
- $\underset{J}{Q} = \underset{Kg}{m} \cdot c \cdot \underset{K}{\Delta T}$ (Heat need to raise-lower the temperature of a substance by ΔT degrees).

Ex. $T_f = 30°C$, $T_{in} = 0°C$ } $\Delta T = 30$ (°C or K)

$T_f = 303K$, $T_{in} = 273K$ } $\Delta T = 303 - 273$, $\Delta T = 30K$

- c = specific heat capacity ($J Kg^{-1} K^{-1}$).

It is the energy needed for a substance to receive in order for its temperature to be increased by 1 kelvin.

Water has a high specific heat capacity (4200 J/Kg*K) and metals in general have low specific heat capacity, since they are good conductors of heat.

$\overline{E_K} = \frac{3}{2} K_B \cdot \Delta T$ (Kinetic Energy of molecules and temperature are proportional quantities)

$\underset{J}{Q} = \underset{Kg}{m} \cdot L \longrightarrow$ Heat required for a substance to experience a change in phase.

L = Latent Heat ($J Kg^{-1}$) → vaporisation, → fusion

When a substance experiences change in phase, there is no temperature change, therefore all the kinetic energy of the molecules becomes potential - elastic energy.

During a change in phase, since there is no temperature change, the kinetic energy of molecules is equal to 0, which means all of the internal energy originates from elastic potential energy, which is generated by the elastic collisions of the particles during a change of phase.

In Thermodynamics in general we mainly talk about ideal gases which means, that we have:
1) Perfect spheres in terms of geometric
2) Discrete forces between them (Van der Waal and Electrostatic)
3) Perfectly Elastic collisions between them or between them and the walls of a "box"

Also: (scalar) $P = \frac{F}{A}$ $\left(\text{pressure} = \frac{\text{force}}{\text{area}}\right)$ $N\,m^{-2}$ or Pascal and $1.013 \cdot 10^{5}$ Pascal $= 1$ atm (atmospheric pressure)

$K_B = \frac{R}{N_A}$ (statistical concept) = Boltzman Constant $= 1.3 \cdot 10^{-23}\ J\,K^{-1}$

$R = 8.31\ J\,mol^{-1}K^{-1}$ (Gas Constant).

S.T.P. = Standard Temperature Pressure (273 Kelvin / or 298 - Pressure is 1atm (atmospheric pressure)),

State Equation $P \cdot V = n \cdot R \cdot T$ or $P \cdot V = \frac{N}{N_A} \cdot R \cdot T \Leftrightarrow P \cdot V = N \cdot K_B \cdot T$

$N_A = 6.02 \cdot 10^{22}$ (avogadro constant)

$n = \frac{N}{N_A}$ or $n = \frac{m}{Mr}$ or $n = \frac{V}{22.7}$ (n = molarity).

In this topic, you will learn about:

- molecular theory for solids, liquids and gases
- density
- the Celsius temperature scale
- the Kelvin temperature scale as a measure of the average kinetic energy of particles
- the internal energy of a system
- phase changes
- specific heat capacity of a substance
- specific latent heat of fusion and vaporization of substances
- conduction, convection and thermal radiation as energy transfer mechanisms
- thermal conductivity
- the Stefan–Boltzmann law
- apparent brightness and luminosity
- the emission spectrum of a black body
- Wien's displacement law.

other parameters of the gas can also change (pressure, density, volume).

Average kinetic energy and the Boltzmann factor

Increased temperatures are linked to increases in the internal energy for a substance. However, we have not made this idea quantitative. The quantity, now known as the Boltzmann constant, was first introduced by Max Planck in a theory that described black-body radiation (you will meet this theory in Topic B.2). However, Planck also linked the Kelvin temperature T of a gas to the average translational kinetic energy E_k of a gas particle by

$$E_k = \frac{3}{2} k_B T$$

The value of k_B is 1.381×10^{-23} J K^{-1}.

One way to think of the Boltzmann constant is as the conversion factor between Kelvin temperature and average kinetic energy of a gas. It reminds us that temperature is the macroscopic measure that we use to assess the amount of kinetic energy in a substance.

converter because it is

total of the kinetic energies of the entities in the system (the atoms and molecules) and the potential energies of the entities.

Thermal energy is a loose term that is generally taken to relate to the movements of the atoms or molecules within an object or system. These movements can be translational, vibrational or rotational.

How can observations of one physical quantity allow for the determination of another? (NOS)

The Boltzmann factor k_B provides a vital link—both numerical and conceptual—between the average kinetic energy of the particles and temperature. Observing one quantity determines the other.

The factor k_B itself is relatively recent and was introduced by Max Planck in 1900. The ideal gas constant R (which you meet in Topic B.3) was all that could be used to link the macroscopic quantities used to describe a gas.

If you study Topic B.4, you will find another interpretation for k_B which follows the link that Boltzmann himself made (even though he never used a fundamental constant in his ideas). The link here is between entropy and probability. So, again, k_B provides the means to determine the value of one quantity when another has been measured.

meausured in Joules per kelvin.

EXAM TIP

You can save yourself time and possible errors if you write this equation, as is, in the equation solver of your graphic display calculator (GDC) and ask the GDC to solve it for you.

WORKED EXAMPLE 7.5

A sample of 120 g of a solid initially at 20 °C is heated by a heater of constant power. The specific heat capacity of the solid is 2500 J kg^{-1} K^{-1}. The temperature of the sample varies with time as shown in Figure 7.6.

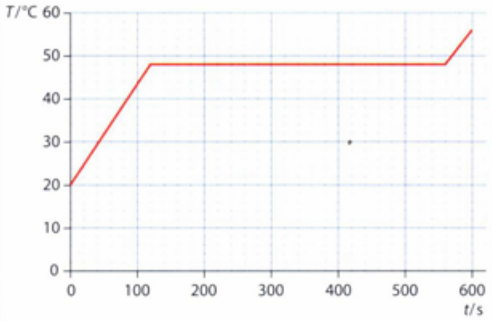
Use the graph to determine:

a the power of the heater

b the melting temperature of the sample

c the specific latent heat of fusion of the sample

d the specific heat capacity of the sample in the liquid phase.



[redacted]

CONTINUED

[redacted]

We already mentioned that a phase change occurs at constant temperature. Ice, for example, melts at 0 °C. For a mass m of ice at 0 °C we need to supply an energy mL which turns the ice into liquid water *without a change in temperature*. How do we explain this observation? At the melting temperature, changing from a solid to a liquid means that the average distance between the molecules increases. Increasing the separation of the molecules requires work (because there are attractive forces between the molecules that need to be overcome). The thermal energy supplied increases the intermolecular potential energy (recall the graph in Figure 7.3) and not the kinetic energy of the molecules. So the temperature stays the same but the internal energy increases since the potential energy increases. It is an interesting question, of course, to ask what if anything *prevents* energy from going into kinetic energy during a phase change.

Consider thermal energy provided to melting ice and water at 0 °C. Microscopically, some thermal energy will inevitably go to a small quantity of water raising its temperature slightly above zero. This slightly warmer water is surrounded by ice and water at 0 °C, and so thermal energy will now be transferred away from this warmer water reducing its temperature back down to

Measuring the specific heat capacity of a liquid

- Tool 1: Understand how to accurately measure mass and temperature to an appropriate level of precision.
- Inquiry 1: Pilot methodologies.
- Inquiry 3: Identify and discuss sources and impacts of random and systematic errors.

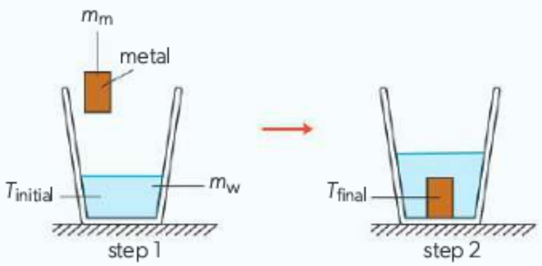
You can use a technique called the **method of mixtures** to measure heat capacities, as shown in Figure 9. Two substances both of known mass but at different initial temperatures are mixed. The final temperature after the mixing is determined. When the specific heat capacity of one substance is known, the energy transferred to the other substance can be determined.

A known mass of water is placed in a container together with a thermometer. A block of metal of known mass m_m and specific heat capacity c_m is placed in boiling water for long enough to ensure that the entire block is at 100 °C. The metal is then removed from the boiling water, quickly dried on paper tissue to remove droplets of water and then immediately transferred to the water in the container. The thermometer is read when the temperature of the mixture has reached its maximum.

The calculation is straightforward:

energy transferred *from* the block = energy transferred *to* the water

$$m_m \times c_m \times (100 - T_{final}) = m_w \times c_w \times (T_{final} - T_{initial})$$



▲ Figure 9 The method of mixtures with a solid and a liquid.

leading to $c_w = \dfrac{m_m \times c_m \times (100 - T_{final})}{m_w \times (T_{final} - T_{initial})}$

where $T_{initial}$ is the starting temperature of the water before the addition of the block and T_{final} is the final temperature of the water and block together.

However, the estimation of T_{final} is not so easy because, in practice, energy is transferred to the surroundings and the container. One way to allow for this is to design the experiment so that $T_{initial}$ begins as far below room temperature as it ends above it. Then, to a fair approximation, the energy transferred into the mixture in the first half of the experiment is equal to the energy transferred out in the second half. A preliminary run is normally required to estimate what m_w needs to be.

Measuring the efficiency of a kettle

- Tool 1: Understand how to accurately measure mass, time, temperature, electric current and electric potential difference to an appropriate level of precision.
- Tool 3: Select and manipulate equations.
- Inquiry 2: Collect and record sufficient relevant quantitative data.

You need a mains power meter that can measure the current and that attaches to a plug socket and an electric kettle (or a portable electric hob and a saucepan).

- Measure the mass of a quantity of water (about 1 kg would be suitable) and put it in the kettle.
- Switch the kettle on until the water boils. Measure the time that this takes. While the water is boiling, note the energy being transferred to the kettle.
- Once the water has boiled, measure the final temperature of the water.
- Determine the energy supplied from the mains. Use the temperature difference, the mass of water and the specific heat capacity of water (c = 4200 J kg^{-1} K^{-1}) calculate the thermal energy transferred to the water. Hence calculate the efficiency of the kettle.

Note: This experiment could also be carried out using a microwave and a bowl of water. The microwave should have a power rating (there may be different settings

Conduction

Imagine a solid with one end kept at a high temperature, as shown in Figure 7.10. The electrons at the hot end of the solid have a high average kinetic energy. This means they move a lot. The moving electrons collide with neighbouring molecules, transferring energy to them and so increasing their average kinetic energy. This means that energy is being transferred from the hot to the cold side of the solid; this is **conduction**.

Through the particles collisions

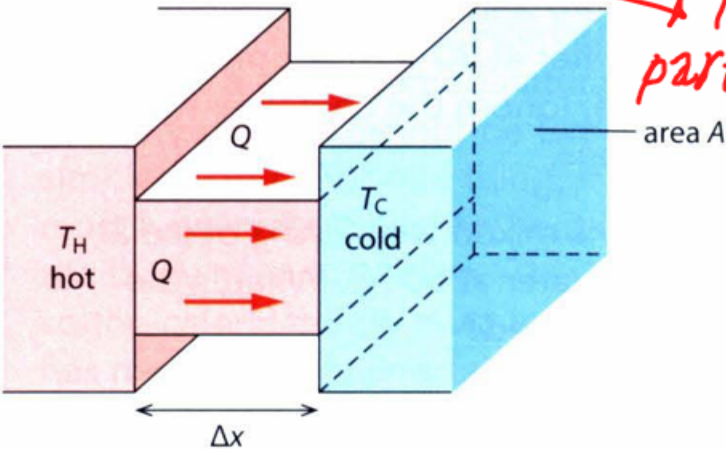


Figure 7.10: Conduction of thermal energy through a solid as a result of a temperature difference.

For a solid of cross-sectional area A, width Δx and temperature difference between its ends ΔT, experiments show that the rate at which energy is being transferred is:

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$ rate of change of thermal energy transfer.

where k is called the conductivity and depends on the nature of the substance. Referring to Figure 7.10, $\Delta T = T_H - T_C$.

$(3k)\ A\frac{T}{L}$

conservat

$kA\frac{100 -}{L}$

$100 -$

Cooling

This is a term that needs care. Throughout this discussion of specific heat capacity and specific latent heat there is a careful distinction between temperature change (heat capacity) and constant temperature conditions (phase change). The term "cooling" only has the meaning of a temperature decrease in physics. In everyday life we are less careful in using the term. On a hot day you might say that you need to "cool down" but what you mean is that you need to transfer more energy from your body to maintain the correct temperature even though you feel "hot". This is a physiological, not a physical, effect.

When water is in the process of freezing to ice, it is not cooling. Its temperature is constant (at 273 K) and energy is being transferred from it.

Should language vary between scientific practice and everyday life?

Thermal energy transfer

Earlier in this topic you saw that an object with a temperature above absolute zero possesses internal energy that is due to two contributions:

- the random motion of its atoms and molecules
- their intermolecular potential energy.

The higher the temperature of the object, the greater the internal energy associated with the molecules.

Energy spontaneously transfers from a region at a high temperature to a region at a low temperature.

There are three ways in which this **thermal energy transfer** can be achieved:

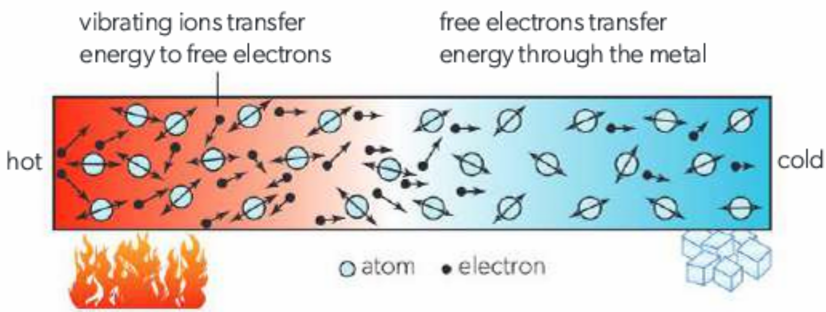
- conduction
- convection
- thermal radiation.

All are important to us on both an individual level and in global terms.

Metals are excellent thermal conductors, just as they are also good electrical conductors. Poor thermal conductors such as glass or some plastics also conduct electricity poorly. This suggests similarities between the mechanisms that lead to both types of conduction. However, you should note that there are still considerable differences in scale between the best metal conductors (copper, gold) and the worst metals (brass, aluminium). There are many laboratory experiments that you can carry out to determine the different thermal properties of good and poor conductors. Figure 13 shows just two demonstrations of conduction.

The use of the words "source" and "sink" link to the same usage in Topic B.4 where they have distinct meanings: source means a provider of energy capable of transferring energy to an absorber (sink) of the energy at a lower temperature.

In conduction processes, energy transfers through the bulk of the material without any large-scale relative movement of the atoms that make up the solid. Thermal conduction and electrical conduction are collectively known as **transport phenomena**.



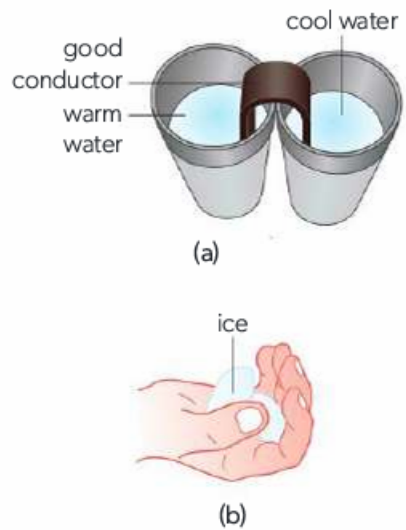
▲ **Figure 14** A good conductor heated to a high temperature at one end soon transfers energy along its length.

Oscillation = Vibration

Atomic vibration occurs in all solids, both metals and non-metals. At all temperatures above 0 K, the ions in the solid have an internal energy. They are vibrating about their average fixed position in the solid. The higher the temperature, the greater is their average kinetic energy, and therefore the higher their mean speed.

Imagine a metal rod heated at one end and cooled at the other (see Figure 14). At the high-temperature end, the ions have a larger average kinetic energy than at the low-temperature end. The ions transfer energy to the free electrons, which re-distribute this energy along the rod. The ion with the smaller energy tends to gain energy, and the other one loses energy in the electron collisions. There is a transfer of internal energy along the metal rod until the whole of the metal rod is at the same temperature.

Conduction can occur in gases and liquids as well as solids. However, the inter-atomic connections are weaker and the gas atoms are about ten times



▲ **Figure 13** Examples of conduction. (a) A good conductor is able to transfer energy from a hot object to a cold one. (b) Ice held in the hand soon chills the fingers.

Thermal conductivity

Conduction is an important factor in the design of many engineering projects. It is important to minimize the transfer of energy from buildings in parts of the world with cold winters. Equally, a good design will maximize the transfer of energy in heat exchangers for power stations of all types. Engineers must be able to quantify the amount of energy conducted through different materials. This is done by defining a quantity known as **thermal conductivity** which is a measure of how good a thermal conductor is at transferring energy through itself when in steady state. **Steady state** is when the temperature at any point in the slab does not change with time. The defining word equation is

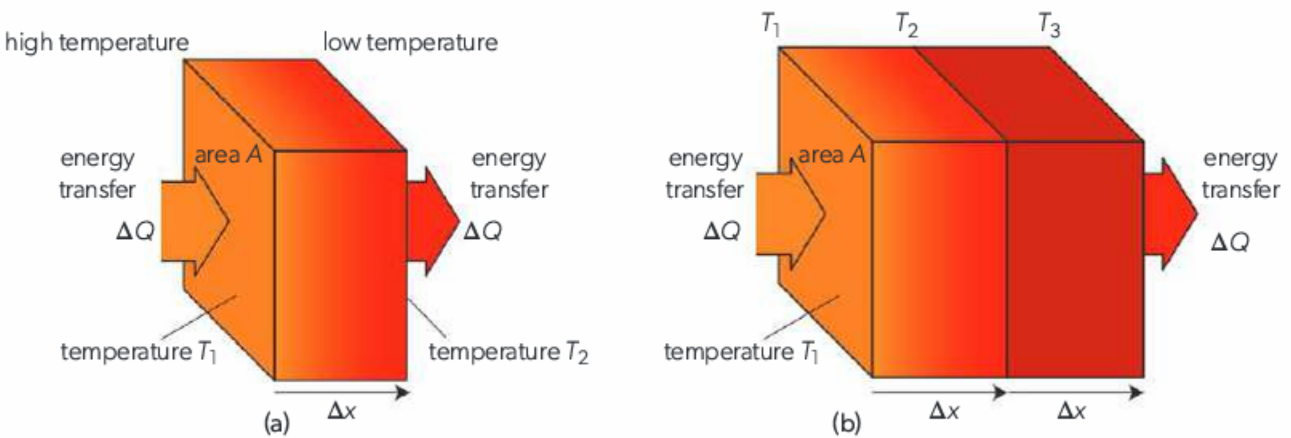
$$\text{thermal conductivity} = \frac{\text{rate of energy transfer}}{\text{area of material} \times \text{temperature gradient across conductor}}$$

or *rate of energy transfer* = *thermal conductivity* × *area of material* × *temperature gradient across conductor*, which in symbols (see Figure 15(a)) is

$$\frac{\Delta Q}{\Delta t} = k \times A \times \frac{\Delta T}{\Delta x}$$

where:

- an energy ΔQ is transferred across the material in a time Δt
- through an area A
- when there is a temperature difference $\Delta T = T_1 - T_2$ across the conductor that has a length Δx.



temperature difference between its ends ΔT, experiments show that the rate at which energy is being transferred is:

$$\boxed{\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}}$$ rate of thermal energy transfer.

where k is called the conductivity and depends on the nature of the substance. Referring to Figure 7.10, $\Delta T = T_H - T_C$.

conservation. Hence

$$kA\frac{100 - T}{L} = (3k)A\frac{T - 0}{L}$$

$$100 - T = 3T$$

$$T = 25\ °C$$

(b) $Q = L \cdot m$

(b) $m = \frac{Q}{L} = \frac{\text{Power} \cdot \text{time}}{L} = \frac{P \cdot t}{L} \Rightarrow$

$$m = \frac{36\cancel{0}\frac{\cancel{J}}{\cancel{s}} \cdot 36\cancel{00}\cancel{s}}{334 \cdot \cancel{10^3}\frac{\cancel{J}}{kg}} = \frac{36^2}{334}\ kg$$

$$\boxed{m = 3.88\ kg.}\ (3.9\ kg)$$

Worked example 10

A layer of ice of a uniform thickness 7.0 cm has formed on the surface of a lake. The temperature of the air above the ice is −12 °C and that of the water below the ice is 0 °C. $A = 1\,m^2$

a. Calculate the rate of thermal energy transfer per unit area through the ice. The thermal conductivity of ice is $2.1\ W\,m^{-1}\,K^{-1}$.

b. Calculate the mass of water that freezes during one hour below one square metre of the ice. The specific latent heat of fusion of ice is $334\ kJ\,kg^{-1}$.

c. Hence calculate, in mm per hour, the rate of change of thickness of the ice. The density of ice is $920\ kg\,m^{-3}$.

(a) $7 \cdot 10^{-2}\,m$ / $T_c = -12\,°C = 273 - 12 = 261\,K$, $T_h = 0\,°C = (0 + 273)\,K = 273\,K$

$$\frac{\Delta Q}{\Delta t} = K \cdot A \cdot \frac{\Delta T}{\Delta x} = 360\ W\ (\text{per } m^2)$$

$$\rho = \frac{m}{V} \Rightarrow 920\frac{\cancel{kg}}{m^3} = \frac{3.9\ \cancel{kg}}{A \cdot d} \Leftrightarrow$$

density = mass / volume

$$\cancel{1}\cancel{A} \cdot d = \frac{3.9}{920}\ m^{\cancel{3}1}$$

$$d = \frac{3.9}{920}\ m = \frac{3\,900}{920}\ mm\ (\text{per hour}) = \boxed{4.24\ mm/h}$$

Convection

Convection is a method of energy transfer that applies mainly to fluids, i.e. gases and liquids. If you put a pan of water on a stove, the water at the bottom of the pan is heated by conduction. As it gets hotter the water expands, it gets less dense and so rises to the top. In this way thermal energy from the bottom of the pan is transferred to the top. Similarly, air over a hot radiator in a room is heated, expands and rises, transferring warm air to the rest of the room. Colder air takes the place of the air that rose and the process repeats, creating **convection currents**. In other words convection is due to differences in density. Convection is responsible for winds in the atmosphere and for ocean currents.

Radiation

Both conduction and convection require a material medium through which thermal energy is to be transferred. The third method of thermal energy transfer, **radiation**, does not. Energy from the sun has been warming the earth for billions of years. This energy arrives at earth as radiation having travelled through the vacuum of space at the speed of light.

Stefan–Boltzmann law

One of the great advances in physics in the nineteenth century was the discovery that all bodies that are kept at some absolute (kelvin) temperature T radiate energy in the form of electromagnetic waves. This is radiation created by oscillating electric charges in the atoms of the body. The power radiated by a body is governed by the **Stefan–Boltzmann law** (obtained experimentally by J. Stefan and deduced theoretically by L. Boltzmann).

KEY POINT

The power radiated per unit area of the body, the radiated **intensity**, is then $\frac{P}{A} = \sigma T^4$.

Strictly, this applies to a theoretical body called a **black body**. This is a perfect radiator as well as a perfect absorber. A black body will absorb all the radiation falling upon it, reflecting none. A black body at low temperature radiates very little, and since it absorbs all the radiation falling on it, it looks black. At high temperature it radiates a lot and looks very bright. A very good example of this is a piece of charcoal: dark when cold, glowing orange-red when hot. (In Chapter 8 we will see how the Stefan–Boltzmann law is modified to apply to real bodies as well.)

Links

Light incident on a shiny metallic surface will reflect with little light being absorbed. Light incident on glass will pass through the glass with very little being absorbed. So why does a black body absorb all the radiation incident on it, transmitting none and reflecting none? The incident light is an electromagnetic wave and so contains an electric field (see Sections 13.3 and 18.1). This field will set electrons in a surface oscillating. In the case of glass, the electrons are very tightly bound to their atoms and so cannot oscillate much. The light just goes through. In the case of the metal, there are lots of free electrons. The electric field makes them oscillate a lot which means they are accelerated. But accelerated charges radiate electromagnetic waves, and this is the reflected light. In a black body, the electric field also accelerates electrons which gain energy, but before they have a chance to radiate they collide with atoms giving them their kinetic energy. The incident radiation has been absorbed and the temperature of the body increases. As the temperature increases more and more the vibrations of the atoms get larger and larger. Now, the collisions between atoms and electrons are very violent

Convection

Convection is the movement of groups of atoms or molecules within fluids (liquids and gases) because of variations in density. Unlike conduction, which involves the microscopic transfer of energy, convection is a bulk property and is described in macroscopic ways. Convection cannot take place in solids. An understanding of convection is important in many areas of physics, astrophysics and geology. In some hot countries, houses are designed to take advantage of natural convection to cool them down in hot weather.

Examples of convection

Figure 18 shows three experiments that involve convection. In all three cases, energy is supplied to a fluid. In Figure 18(a), a candle heats the air underneath a tube (a chimney) that leads vertically out of the box. The air molecules immediately above the flame move further apart decreasing the air density in this region. With a reduced density compared with the surrounding air, these molecules experience an upthrust and move up through the left-hand chimney.

This upward air movement reduces the pressure in the box slightly and causes cooler air to be pulled down the right-hand chimney. Further heating of the air above the flame leads to a continuous current of cold air down the right-hand chimney and hot air up the left-hand tube. This is a **convection current**.

Similar currents can be demonstrated in liquids. Figure 18(b) shows a small crystal of a soluble dye (potassium permanganate, $KMnO_4$) placed at the bottom of a beaker of water. When the base of the beaker is heated gently near to the crystal, water at the base heats, expands becoming less dense, and rises.

There is also a convection current in Figure 18(c) where a glass tube, in the shape of a rectangle, again with a small soluble coloured crystal in the tube, can sustain a convection current that moves all around the tube.

A convection current is the mechanism through which all the water heated in a saucepan eventually reaches a uniform temperature. There are many examples of convection in action. Figure 19 shows examples from the natural world. There are many others.

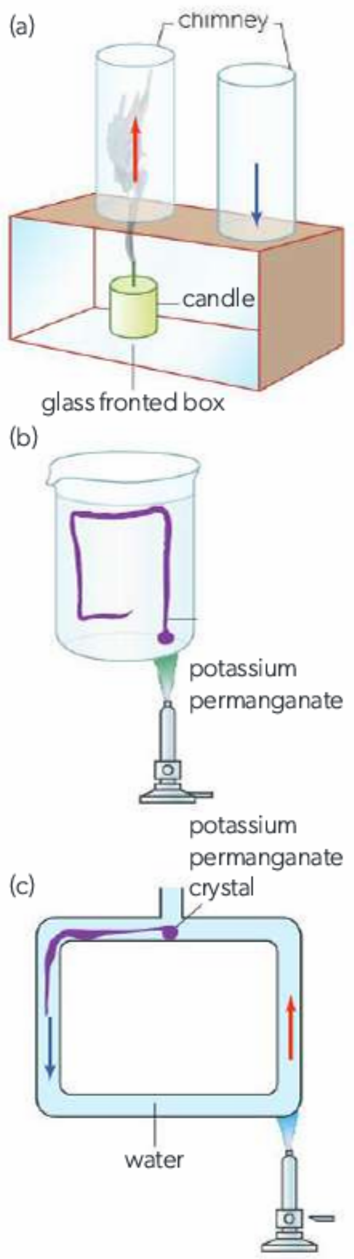


Figure 7.13 shows three black-body spectra emitted from the same surface at three different temperatures of the surface (T = 273 K, 320 K and 350 K). We see that with increasing temperature, the peak of the curve occurs at lower wavelengths and the height of the peak increases. The quantity B plotted on the vertical axis is the power emitted per unit area per unit wavelength and so has units of $(\mathrm{W\,m^{-2}})\,\mathrm{m^{-1}} = \mathrm{W\,m^{-3}}$. It is called spectral intensity. The area under the graph is the radiated intensity, σT^4.

$B = \frac{W}{m^3}$

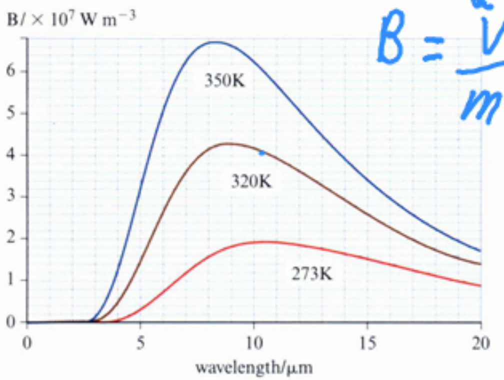


Figure 7.13: Black-body spectra for a body at the three temperatures shown. Notice that as the temperature increases the peak gets higher and the wavelength at the peak shifts to the left.

The energy radiated is electromagnetic radiation and is distributed over an infinite range of wavelengths. The peak of the spectrum corresponds to a specific wavelength λ_{max}, Figure 7.14, called the **peak wavelength**.

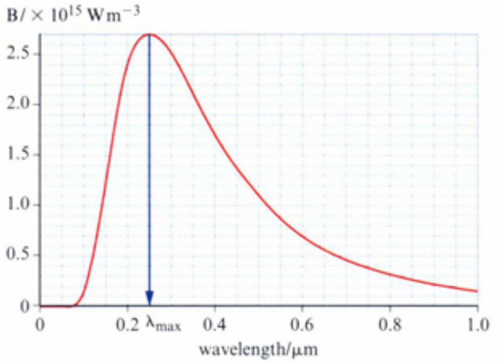


Figure 7.14: For this graph, the peak wavelength is 0.25×10^{-6} m.

Wien's law

The peak wavelength λ_0 depends on temperature according to **Wien's law**.

Wien's law states that $\lambda_{max} T = 2.90 \times 10^{-3}$ K m.

So, for the graph of Figure 7.14, the temperature is $T = \frac{2.90 \times 10^{-3}}{0.25 \times 10^{-6}} = 11600 \approx 1.2 \times 10^4$ K.

EXAM TIP

Increasing the temperature shifts the peak wavelength to the left and makes the curve taller.

CHECK YOURSELF 6

The area of the body in Figure 7.14 is doubled. How does the graph change?

WORKED EXAMPLE 7.7

A human body has temperature 37 °C, the average earth surface temperature is 288 K and the temperature of the sun is 5800 K. In each case, calculate the peak wavelength of the emitted radiation.

Answer

We just have to apply Wien's law, $\lambda_{max} T = 2.90 \times 10^{-3}$ K m, and make sure we use kelvins in each case. So:

human body: $\lambda_{max} = \frac{2.90 \times 10^{-3}}{273 + 37} \approx 9 \times 10^{-6}$ m, an infrared wavelength

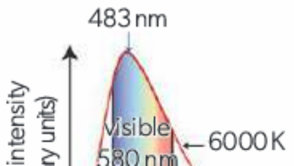
earth surface: $\lambda_{max} = \frac{2.90 \times 10^{-3}}{288} \approx 1 \times 10^{-5}$ m, an infrared wavelength

sun: $\lambda_{max} = \frac{2.90 \times 10^{-3}}{5800} \approx 5 \times 10^{-7}$ m, which is a visible wavelength.

Global impact of science — The potter's kiln

Fabrication of ceramic objects was an early technology developed by humans.

A potter needs to know the temperature of the inside of a kiln while the clay is being "fired" to transform it into porcelain. Some potters simply view the interior of the kiln through a small hole. They can tell by experience what the temperature is from the emitted colour of the pots inside. Other potters use an instrument called a pyrometer. A tungsten filament is placed at the entrance to the kiln between the kiln interior and the potter's eye. An electric current is supplied to the filament, and this is increased until the filament disappears by merging into the background. At this point the filament is at the same temperature as the interior of the kiln. The filament system will have previously been calibrated so that the current required for the filament to disappear can be equated to the filament temperature.



The emission spectrum from a black body

Although there is a predominant colour to the radiation emitted from a black-body radiator, this does not mean that only one wavelength emerges. To study the whole of the radiation that the black body emits, an instrument called a spectrometer is used. It measures the intensity of the radiation at a particular wavelength across a range of wavelengths.

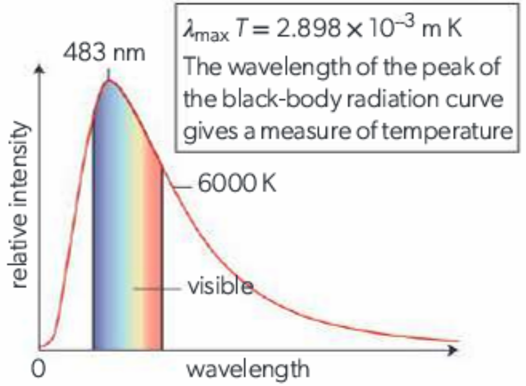
Intensity is the power emitted per square metre. As an equation, this is

$$I = \frac{P}{A}$$

Intensity = $\frac{\text{power}}{\text{area}}$

where I is the intensity, P is the power emitted and A is the area on which the power is incident. The units of intensity are $\mathrm{W\,m^{-2}}$ or $\mathrm{J\,s^{-1}\,m^{-2}}$.

A typical intensity–wavelength graph is shown in Figure 24 for a black body at the temperature of the visible surface of the Sun, about 5800 K. The Sun can be considered as a near-perfect black-body radiator. The graph shows how the relative intensity of the radiation varies with the wavelength of the radiation at which the intensity is measured. No scale is given on either axis. The graph shows relative values of intensity.



▲ **Figure 24** The spectrum of the Sun assuming that it is a black-body radiator.

Important features of this graph:

- There is a peak value at about 500 nm (somewhere between green and blue light to our eyes). Is it a coincidence that the human eye has a maximum sensitivity in this region or is this biological evolution at work?
- There are significant radiations at all visible wavelengths.
- There is a steep rise from zero intensity. Notice that the line does not quite go through the origin.

Figure 7.13 shows three black-body spectra emitted from the same surface at three different temperatures of the surface (T = 273 K, 320 K and 350 K). We see that with increasing temperature, the peak of the curve occurs at lower wavelengths and the height of the peak increases. The quantity B plotted on the vertical axis is the power emitted per unit area per unit wavelength and so has units of $(\mathrm{W\,m^{-2}})\,\mathrm{m^{-1}} = \mathrm{W\,m^{-3}}$. It is called spectral intensity. The area under the graph is the radiated intensity, σT^4.

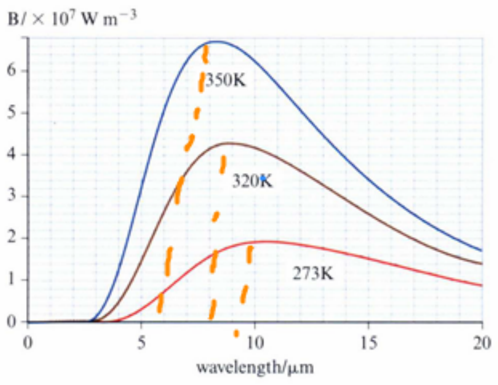


Figure 7.13: Black-body spectra for a body at the three temperatures shown. Notice that as the temperature increases the peak gets higher and the wavelength at the peak shifts to the left.

The energy radiated is electromagnetic radiation and is distributed over an infinite range of wavelengths. The peak of the spectrum corresponds to a specific wavelength λ_{max}, Figure 7.14, called the peak wavelength.

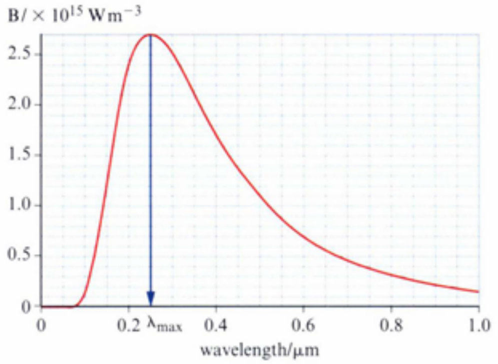


Figure 7.14: For this graph, the peak wavelength is 0.25×10^{-6} m.

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WORKED EXAMPLE 7.7

A human body has temperature 37 °C, the average earth surface temperature is 288 K and the temperature of the sun is 5800 K. In each case, calculate the peak wavelength of the emitted radiation.

[redacted]

GDC Sol

$\lambda_{max} \cdot (273 + 37) = 2.9 \cdot 10^{-3}$ m·K

$\lambda_{max} = 9.35 \cdot 10^{-6}$ m

This family of curves tells us that, as **temperature increases**:

- at each wavelength, the **overall intensity increases** (because the curve is higher)
- the **total power emitted per square metre increases** (because the total area under the curve is greater)
- the curves skew towards shorter wavelengths (higher frequencies)
- the peak of the curve moves to shorter wavelengths.

The next step is to focus on the exact changes between these curves.

It goes to the left

Wien's displacement law

In 1893, Wilhelm Wien was able to deduce the way in which the shape of the black-body emission graph depends on temperature. He showed that the height of the curve and the overall width depends on temperature alone. His full law allows predictions about the height of any point on the curve, but you will only use it to predict the peak of the intensity curve.

Wien's displacement law states that the wavelength at which the intensity is a maximum λ_{max} is related to the absolute temperature of the black body T by:

$$\lambda_{max} T = b$$

where b is known as **Wien's displacement constant** which has the value 2.9×10^{-3} m K.

Units for *b*

Notice that the unit for b is metre kelvin, mK, and must be written with a space between the symbols. Take care not to write it as mK which means millikelvin.

Stefan–Boltzmann law

The scientists Stefan and Boltzmann independently derived an equation that predicts the total power radiated from a black body at a particular temperature. The law applies across all the wavelengths that are radiated by the black body. Stefan derived the law empirically in 1879 and Boltzmann produced the same law theoretically five years later.

The **Stefan–Boltzmann law** states that the total power (luminosity) L radiated by a black body is given by the equation

$$L = \sigma A T^4$$

where A is the total surface area of the black body and T is the absolute temperature of the surface. The constant σ is known as the Stefan–Boltzmann constant and has the value $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. The law refers to the total power radiated by the object, but this is the same as the energy radiated per second. It is easy to show that the energy radiated each second by one square metre of a black body (so $A = 1$) is σT^4. This variant of the full law is known as Stefan's law.

The unit of L is the watt ($\mathrm{W} \equiv \mathrm{J\,s^{-1}}$).

What applications does the Stefan–Boltzmann equation have in astrophysics and in the use of solar energy?

You will meet the Stefan–Boltzmann equation a number of times in this course. It is used in astrophysics for the calculation of the properties of individual stars. When applied to our Sun, it allows us to reach conclusions about the energy reaching the top of Earth's atmosphere and therefore make climate models of our planet.

Luminosity and apparent brightness

In an example of lack of economy of symbols and terms, the quantity $I = \frac{P}{4\pi d^2}$ is called **apparent brightness** by astronomers and is given the symbol b. Thus, apparent brightness is the power received per unit area of the detector. Astronomers also call the total power radiated by a star **luminosity**, L and so in astronomy we have:

$b = \frac{L}{4\pi d^2}$ = what you receive.

Luminosity is a major characteristic of a star. As we saw when we discussed black-body radiation, stars are excellent approximations to a black body and so the luminosity of a star is given by the Stefan–Boltzmann law:

$L = \sigma A T^4$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant, A is the surface area of the star and T its surface temperature in kelvin.

Now, two stars of the same luminosity will not appear equally bright when observed from earth if they are at different distances from earth. Apparent brightness is a measure of how bright a star actually appears.

By combining the formula for luminosity with that of apparent brightness we see that

$b = \frac{\sigma A T^4}{4\pi d^2}$

Apparent brightness is easily measured (with a charge-coupled device, or CCD). If we also know the luminosity then we can determine the distance.

WORKED EXAMPLE 7.9

The apparent brightness of a star is 3.4×10^{-8} W m^{-2} and its luminosity is 5.1×10^{28} W. What is its distance?

$L = 5.1 \cdot 10^{28}$

$b = 3.4 \cdot 10^{-8}$ Wm^{-2}

$\boxed{d = 3.5 \cdot 10^{17} \text{ m}}$

WORKED EXAMPLE 7.10

$r_A = 3 r_B$ . .

a The radius of star A is three times that of star B and its temperature is double that of B. Find the ratio of the luminosity of A to that of B.

b The stars in **a** have the same apparent brightness when viewed from earth. Calculate the ratio of their distances.

[redacted]

$V_{sp.}(r) = \frac{4}{3}\pi \cdot r^3$

$\frac{dV}{dr} = A(r) = \left(\frac{4}{3}\pi r^3\right)' =$

$V'(r) = A(r) = \frac{4}{3}\pi \cdot (r^3)'$

$A(r) = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2$

$\boxed{A(r) = 4\pi r^2}$

surface area of a planet.

WORKED EXAMPLE 7.11

The apparent brightness of a star is 6.4×10^{-8} W m^{-2}. Its distance is 15 ly (1 ly = 9.46×10^{15} m). Find its luminosity.

Answer

We use $b = \frac{L}{4\pi d^2}$ to find

$L = b 4\pi d^2$

$= \left(6.4 \times 10^{-8} \frac{\text{W}}{\text{m}^2}\right) \times 4\pi \times (15 \times 9.46 \times 10^{15})^2 \text{ m}^2$

$= 1.62 \times 10^{28}$ W $\approx 1.6 \times 10^{28}$ W

Observations — Discovering galaxies

▲ **Figure 28** The M31 galaxy.

The concept of a galaxy did not exist until the early 20th century. In 1925, Edwin Hubble deduced that M31 was 900 000 light years away and therefore well beyond the edge of our own galaxy. This enabled Hubble to use the apparent brightness of M31 to deduce that its mass was about $3.5 \times 10^9 M_\odot$ and its luminosity about $7 \times 10^8 L_\odot$.

It was later discovered that M31 (the Andromeda galaxy) is in fact 2.5 million light years away. Its greater distance means that it is much brighter than Hubble thought. The luminosity of the andromeda galaxy is now thought to be $2.6 \times 10^{10} L_\odot$.

Combining the Stefan–Boltzmann law with the equation for the area of a sphere of radius R, assuming that these stars are spherical in shape, gives $L = \sigma 4\pi R^2 T^4$. The luminosity of a star depends on its temperature and its radius.

Topic E.5 takes this relationship further to discuss an important classification tool in stellar astronomy—the Hertzsprung–Russell diagram.

$\frac{9\cancel{r_B^2} \cdot 16 \cdot \cancel{T_B^4}}{\cancel{r_B^2} \cdot \cancel{T_B^4}} = 9 \cdot 16 = 144.$

Practice questions

HW

23. The star Antares has a luminosity of $76\,000 L_\odot$ ($L_\odot = 3.83 \times 10^{26}$ W) and is at a distance of 5.2×10^{18} m from Earth.

Calculate the apparent brightness of Antares as seen from Earth.

24. The minimum intensity of light that can be detected by unaided human eye is approximately 10^{-10} W m^{-2}.

Estimate the distance at which the Sun could just be seen by unaided eye.

25. A sphere of radius R has an absolute temperature T and it radiates power P. A second sphere has a radius $2R$. The spheres can be assumed to be black-body radiators. Determine:

a. the power radiated by the second sphere, when its absolute temperature is T

b. the absolute temperature of the second sphere, when it radiates power P.

$\frac{L_A}{L_B} = \frac{\cancel{\sigma} \cdot A_A \cdot T_A^4}{\cancel{\sigma} \cdot A_B \cdot T_B^4} = \frac{\cancel{4\pi} \cdot r_A^2 \cdot T_A^4}{\cancel{4\pi} \cdot r_B^2 \cdot T_B^4} = \frac{(3r_B)^2 \cdot (2T_B)^4}{r_B^2 \cdot T_B^4} =$

WORKED EXAMPLE 7.10

a The radius of star A is three times that of star B and its temperature is double that of B. Find the ratio of the luminosity of A to that of B.

b The stars in **a** have the same apparent brightness when viewed from earth. Calculate the ratio of their distances.

Answer

a $$\frac{L_A}{L_B} = \frac{\sigma 4\pi R_A^2 T_A^4}{\sigma 4\pi R_B^2 T_B^4} = \frac{R_A^2 T_A^4}{R_B^2 T_B^4} = \frac{(3R_B)^2(2T_B)^4}{R_B^2 T_B^4} = 3^2 \times 2^4 = 144$$

b $$\frac{b_A}{b_B} = 1$$

$$1 = \frac{\left(\frac{L_A}{4\pi d_A^2}\right)}{\left(\frac{L_B}{4\pi d_B^2}\right)}$$

$$1 = \left(\frac{L_A}{L_B}\right)\left(\frac{d_B^2}{d_A^2}\right)$$

$$\Rightarrow \frac{d_A}{d_B} = \sqrt{\frac{L_A}{L_B}} = \sqrt{144} = 12$$

$$b_A = b_B \iff$$

$$\left(\sqrt{}\right)^{\cancel{2}} = \left(\sqrt{}\right)^{\cancel{2}}$$

$$\frac{L_A}{\cancel{4\pi} d_A^2} = \frac{L_B}{\cancel{4\pi} \cdot d_B^2}$$

WORKED EXAMPLE 7.11

The apparent brightness of a star is 6.4×10^{-8} W m^{-2}. Its distance is 15 ly (1 ly $= 9.46 \times 10^{15}$ m). Find its luminosity.

Answer

We use $b = \frac{L}{4\pi d^2}$ to find

$$L = b 4\pi d^2$$

$$= \left(6.4 \times 10^{-8} \frac{\text{W}}{\text{m}^2}\right) \times 4\pi \times (15 \times 9.46 \times 10^{15})^2 \text{ m}^2$$

$$= 1.62 \times 10^{28} \text{ W} \approx 1.6 \times 10^{28} \text{ W}$$

$$\sqrt{\frac{L_A}{L_B}} = \sqrt{\left(\frac{d_A}{d_B}\right)^{\cancel{2}}} \iff \frac{d_A}{d_B} = \sqrt{144} = 12.$$

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from Earth.

Calculate the apparent brightness of Antare
from Earth.

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unaided human eye is approximately 10^{-10} W

Estimate the distance at which the Sun coul
seen by unaided eye.

$$\boxed{T_f = 92.3\,^\circ C}$$

(20) $Q_{gained} + Q_{lost} = 0$

$$- m_s \cdot L_v - m_s \cdot C_w \cdot (-50) + 0.2 \cdot L_f + 0.2 \cdot 4200 \cdot 50 = 0$$

$m_s = 0.05307\ Kg \Rightarrow$

$$\boxed{m_s = 53.1 g}$$

(18) $Q_{gained} + Q_{lost} = 0\ J \Leftrightarrow$

$$m_{ice} \cdot L_f + m_{ice} \cdot C_w \cdot \Delta T_{ice} + M_w \cdot C_w \cdot \Delta T_w = 0 \Leftrightarrow$$

10

(the ice has transformed into water)

$$m_{ice} \cdot 334 \cdot \cancel{10^3} + m_{ice} \cdot 4{,}2\cancel{00} \cdot 10 + 1 \cdot 4{,}2\cancel{00} \cdot (-1\cancel{0}) = 0 \Leftrightarrow$$

$$334 \cdot m_{ice} + 42 \cdot m_{ice} - 42 = 0 \Leftrightarrow$$

$$\frac{\cancel{376} \cdot m_{ice}}{\cancel{376}} = \frac{42}{376} \Leftrightarrow m_{ice} = 0.112\ Kg$$

18 Ice at 0 °C is added to 1.0 kg of water at 20 °C, cooling it down to 10 °C. Determine how much ice was added.

19 A quantity of 100 g of ice at 0 °C and 50 g of steam at 100 °C are added to a container that has 150 g water at 30 °C. Determine the final temperature. Ignore the container itself in your calculations.

20 Steam at 100 °C is mixed with 200 g of ice at 0 °C to produce water at 50 °C. What is the mass of steam required?

21 Two bodies (same mass and initial temperature) are dropped into two identical containers filled with water. The mass and temperature of the water in both containers is the same. The graphs in Figure 7.9 show the cooling curves of the two bodies.

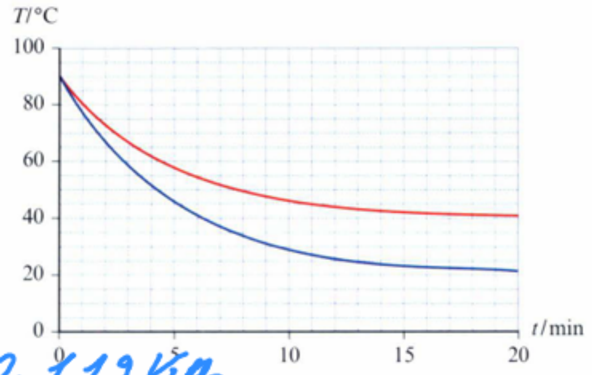

Figure 7.9: For question 21.

Which body has the higher specific heat capacity?

$Q_s + Q_{ice} + Q_w = 0 \Leftrightarrow$

$$0.05 \cdot 2.26 \cdot 10^6 - 0.05 \cdot 4{,}200 \cdot (T_f - 373) +$$
$$0.1 \cdot 334 \cdot 10^3 + 0.1 \cdot 4200 (T_f - 273) +$$
$$0.15 \cdot 4200 \cdot (T_f - 303) = 0$$

$$5 \cdot 22{,}600 + 210 \cdot (T_f - 373) +$$
$$33{,}400 + 420 \cdot (T_f - 273) +$$
$$630 \cdot (T_f - 303) = 0 \Leftrightarrow$$

$$113{,}000 + 210 T_f - 210 \cdot 373 +$$
$$33{,}400 + 420 T_f - 420 \cdot 273 +$$
$$630 T_f - 630 \cdot 303 = 0 \Leftrightarrow$$

$\boxed{\Delta t = 7.6\ s}$

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

$$L = \sigma A T^4$$

apparent brightness → $b = \frac{L}{4\pi d^2}$

what you receive.

$\lambda_{max} T = 2.9$[redacted]

$$\frac{\Delta Q(\cancel{J})}{14\pi\ \cancel{J}/s} = \Delta t$$

$$\Rightarrow \Delta t = \frac{334}{14\pi}$$

CONTINUED

34 A cylindrical metallic rod has radius 1.0 cm and length 25 cm. One end of the rod is placed into a large chunk of ice at 0 °C and the other in boiling water at 100 °C. The thermal conductivity of the rod is 350 W m⁻¹ K⁻¹. Estimate how long it will take to melt 1 g of ice. It takes about 334 J to melt 1 g of ice at 0 °C.

same mass

35 Calculate the ratio of the power radiated per unit area from two black bodies at temperatures 900 K and 300 K.

(34) $r = 10^{-2}\,m$ $\quad 10^{-2}\,m = r$ $\quad \ell = 25\cdot 10^{-2}\,m$ $\quad K = 350\ W m^{-1} K^{-1}$

$T_c = 273\,K$ / $T_h = 373\,K$ $\quad m = 10^{-3}\,Kg$

$334\,J(10^{-3}\,Kg)$

$$\frac{\Delta Q}{\Delta t} = 14\pi\ J/s$$

$$\frac{\Delta Q}{\Delta t} = K\cdot A\cdot\frac{\Delta T}{\Delta x} \Rightarrow$$

$$\frac{\Delta Q}{\Delta t} = 350\ W \cancel{m^{-1}}\cancel{K^{-1}}\cdot\frac{100\cancel{K}}{25\cdot 10^{-2}\cancel{m}}\cdot\pi\cdot(10^{-2})^2\cancel{m^2}$$

$$\frac{\Delta Q}{\Delta t} = \frac{350\cdot\cancel{100}\cdot\pi\cdot\cancel{10^{-4}}}{25}\cdot\cancel{10^2}$$

(35) P (or L) =

L = luminosity (= is the power radiated)

$$\frac{P_{300}}{P_{900}} = \frac{L_{300}}{L_{900}} = \frac{\cancel{\sigma}\cdot\cancel{A}\cdot T_{300}^4}{\cancel{\sigma}\cdot\cancel{A}\cdot T_{900}^4} = \frac{\cancel{300^4}\ 1}{\cancel{300^4}\cdot 3^4} = \frac{1}{81}$$

39 The surface area and kelvin temperature of a black body are both doubled. By what factor do:

a the radiated power

b the radiated intensity

increase?

$$\boxed{\frac{P_{300}}{P_{900}} = \frac{1}{81}}$$

(39) (a) $P_{new} = \sigma\cdot 2A\cdot 16T^4 = 32\cdot A\cdot T^4\cdot\sigma = 32\cdot P_{old}$

(b) $I_{new} = \frac{P_{new}\ (\text{or } L)}{A_{new}} = 16\cdot I_{old}.$